# AN OPTIMUM LOCAL GEOID MODEL FOR IRAN BASED ON THE LEAST-SQUARES MODIFICATION OF STOKES'S FORMULA

Dr. Ramin Kiamehr

Department of Geodesy and Geomatics, Zanjan University, 45195-313, Zanjan, Iran,
Tel: +98 241 515 2224, Fax: +98 241 528 3204, Email: ramin@znu.ac.ir

## ABSTRACT

The modification of Stokes's formula allows the user to compensate the lack of a global coverage of gravity data by a combination of terrestrial gravity and a global geopotential model. The minimization of the errors of truncation gravity data and potential coefficients could be treated in a least-squares sense as is the basic ingredient in the Royal Institute of Technology (KTH) approach as proposed by Sjöberg in 1984. This article presents the results from a recent research, whose main purpose is to evaluate the KTH approach numerically and to compute a gravimetric geoid model for Iran. The new geoid model (IRG04) was computed based on the least-squares modification of Stokes's formula, the GRACE global geopotential model, a high-resolution digital terrain model and the NKG gravity anomaly database. The IRG04 was fitted to 247 GPS/levelling points by a 7-parameter transformation, yielding an all-over fit of 43 cm and 3.5 ppm. Also, we found a significant expected difference between the IRG04 and TUG06 models in rough topographic areas (up to ± 4 m). As the major ground data and global geopotential model were almost same in the two models, we believe that there are different reasons that come into play for interpreting the discrepancies between them, as the method for eliminating outliers from the gravity database, the interpolated denser gravity observations using the high-resolution digital elevation model before Stokes's integration, the potential of the LSM kernel, which matches the errors of the terrestrial gravity data, GGM and the truncation error in an optimum way, and the effect of applying more precise correction terms in the KTH approach compared to the remove-compute-restore method. It is concluded that the least-squares modification method with additive corrections is a very promising alternative for geoid computation.

**Keywords:** Gravity database, least-squares modification of Stokes's function, regional geoid determination, SRTM, GRACE, GPS/levelling, Iran

## 1. INTRODUCTION

During the last 15 years a number of geoid models have been computed by different Working Group for Geoid Determination of the Iran, the four latest models Hamesh and Zomorrodian 1992 (IfAG), Najafi 2004 (KNTUG) and Safari at el., 2005 (TUG05), have all been derived using different techniques. The IfAG geoid model used the remove-compute-restore (r-c-r) method utilising the Fast Fourier Transform (FFT) to speed up the calculations; see e.g. Forsberg (1999) and Forsberg et al. (2004). Also, KTNUG and TUG05 geoid models computed based on the Helmert Scheme (Vanicek at el. 1995) and ellipsoidal Bruns formula, respectively.

Today the overall goal of the NCC Working Group for Geoid Determination is to compute a precise geoid model for Iran namely 1-cm (1 sigma) in smooth and moderate topography and a decimetre level in rough and mountainous areas. To achieve this, it has been deemed necessary to improve the applied theory and to investigate alternative computation techniques.

One such technique is the least-squares modification method with additive corrections, which has been developed at the Royal Institute of Technology (KTH) in Stockholm; see Sjöberg (1984), (1991), (2003b), Ågren (2004a), and Kiamehr (2006b). It includes least-squares modification of Stokes' kernel and separate (additive) corrections for the effects of topography, downward continuation, atmosphere and the ellipsoidal shape of the Earth. This method was successfully applied in the determination of several regional geoid models in Zambia (Nsombo 1996), Ethiopia (Hunegnaw 2001), the Baltic countries (Ellmann 2004), Iran (Kiamehr 2006a) and Iran (Ågren at el. 2006).

The main purpose of this research is to present the results from an on-going research in Zanjan University. Important aims of the project are to evaluate the least-squares modification method with additive corrections numerically and to compute a precise gravimetric geoid model over Iran. The project is also meant to contribute to the National Cartographic Centre of Iran (NCC) efforts to evaluate different geoid computation methods.

The paper starts with a brief review of the computational scheme, which is based on the Least-Squares Modification of Stokes's (LSMS) formula with additive correction terms (Sjöberg 2003c).We explain the procedure to create and evaluate the gravity database, Global Geopotential Model (GGM) and Digital Elevation Model (DEM), as well as other parameters applied in the construction of the new Swedish gravimetric geoid model (IRG04). Finally, we evaluate and compare IRG04 with respect to the recent TUG06 model using GPS/levelling data.

## 2. GEOID DETERMINATION BASED ON THE LEAST-SQUARES MODIFICATION OF THE STOKES'S FORMULA WITH ADDITIVE CORRECTIONS

In the least-squares modification of Stokes's formula (e.g. Sjöberg 1991), Stokes's kernel is modified in such a way that the expected global mean square error is minimised. This technique can be applied with the standard r-c-r estimator (e.g. Ågren 2004a), but according to the KTH practice, the so-called *combined estimator* is preferred (Sjöberg 2003c). This means that Stokes' formula (truncated to a cap) is applied to the uncorrected surface gravity anomaly, $\Delta g$ . After that, the geoid height $N$ is computed by adding a number of corrections, i.e.

$$N = \frac{R}{4\pi\gamma}\iint_{\sigma_0} S^L(\psi)\Delta g d\sigma + \frac{R}{2\gamma}\sum_{n=2}^{M}\left(s_n + Q_n^L\right)\Delta g_n^{GGM} + \delta N_{comb}^{Topo} + \delta N_{DWC} + \delta N_{comb}^{a} + \delta N_e, \qquad (1)$$

where $\sigma_0$ is the spherical cap, $R$ is the mean Earth radius, $\gamma_0$ is mean normal gravity, $S^L(\psi)$ is the modified Stokes's function, $s_n$ are the modification parameters, $M$ is the maximum degree of the Global Geopotential Model (GGM), $Q_n^L$ are the Molodensky truncation coefficients and $\Delta g_n^{GGM}$ is the Laplace harmonic of the gravity anomaly determined by the GGM of degree $n$. The upper limit $L$ is arbitrary and generally not equal to $M$.

The four *additive corrections* of Eq. (1) are derived in such a way that the same result is ideally obtained as when the remove-compute-restore technique is utilised (except for numerical effects). They can be computed as:

- The *combined topographic effect* $\delta N_{comb}^{Topo}$ (Sjöberg and Nahavandchi 2000 and Sjöberg 2006) can be computed by:

$$\delta N_{comb}^{Topo} = -\frac{2\pi G\rho}{\gamma}[H^2 + \frac{2}{3R}H^3], \tag{2}$$

where $\rho$ is the mean topographic mass density and $H$ is the orthometric height and $R$ is the mean radius of the Earth. Sjöberg (2006b) proves that Eq. (2) is exact.

- The *downward continuation effect* $\delta N_{DWC}$ is (Sjöberg 2003a; Ågren 2004b),

  $$\delta N_{DWC}(P) = 3\frac{N_P^0}{r_P}H_P + \frac{R}{2\pi}\sum_{n=2}^{M}\left(s_n + Q_n^M\right)\left[\left(\frac{R}{r_P}\right)^{n+2} - 1\right]\Delta g_n^{GGM}(P) + \frac{R}{4\pi\gamma}\iint_{\sigma_0} S^L(\psi)\left(\left.\frac{\partial \Delta g}{\partial r}\right|_Q \left(H_P - H_Q\right)\right)d\sigma_Q, \tag{3}$$

  where $P$ is the computation point and $r_P = R + H_P$, $N_P^0$ is an approximate value of the geoidal height and $Q$ is the running point in Stokes's integral.
- The *combined atmospheric effect* $\delta N_{comb}^a$ can be approximated to order $H$ by (Sjöberg and Nahavandchi 2000)

  $$\delta N_{comb}^a(P) = -\frac{2\pi R\rho_0}{\gamma}\sum_{n=2}^{\mathrm{M}}\left(\frac{2}{n-1} - s_n - Q_n^M\right)H_n(P) - -\frac{2\pi R\rho_0}{\gamma}\sum_{n=M+1}^{\infty}\left(\frac{2}{n-1} - \frac{n+2}{2n+1}Q_n^M\right)H_n(P), \tag{4}$$

  where $\rho_0$ is the atmospheric density at sea level, $H_n$ is the Laplace harmonic of degree $n$ for the topographic height and $M$ is the maximum degree of the GGM.
- The *ellipsoidal correction to the modified Stokes's formula* $\delta N_e$ to order $e^2$ is (Sjöberg 2004):

$$\delta N_e(P) = \frac{R}{2\gamma}\sum_{n=2}^{\infty}\left(\frac{2}{n-1} - s_n^* - Q_n^M\right)\cdot\left(\frac{a-R}{R}\Delta g_n^{GGM}(P) + \frac{a}{R}(\delta g_e)_n\right), \tag{5}$$

where $s_n^* = s_n$ if $2 \le n \le M$ and $s_n^* = 0$ otherwise. Furthermore,

$$(\delta g_e)_n = \frac{e^2}{2a}\sum_{m=-n}^{n}\left\{\left[3 - (n+2)F_{nm}\right]T_{nm} - (n+1)G_{nm}T_{n-2,m} - (n+7)E_{nm}T_{n+2,m}\right\}Y_{nm}(P), \tag{6}$$

in which $T_{nm}$ are spherical harmonic coefficients for the disturbing potential. See Sjöberg (2004) for the ellipsoidal coefficients $E_{nm}$, $F_{nm}$ and $G_{nm}$.

One problem with using the combined geoid estimator in Eq. (1) is that Stokes's quadrature is made on the rough surface gravity anomaly, which results in large *discretisation errors*. However, by taking advantage of the r-c-r philosophy for the gridding of a comparatively dense gravity anomaly grid using a smoothing topographic correction, such errors can be counteracted; see Sjöberg (2003b) and Ågren (2004a). This makes it possible to take advantage of the high-frequency information available in the DEM. A practical drawback here is that dense grids are required in rough mountain areas, which can be cumbersome. Some advantages with the combined estimators are that the "real" importance of the correction terms is apparent and that it is easier to compute the atmospheric and ellipsoidal corrections in this way. One also avoids the global quadrature required to compute the direct and indirect topographic effects when the remove-compute-restore estimator is used.Choosing a suitable GGM and modification parameters $s_n$ are essential steps in the determination of a geoid model using the LSMS formula. We need to estimate the signal and error degree variances of the GGM and terrestrial gravity to be able to estimate the least-squares modification coefficients. The signal degree variances for the degrees above the GGM are generated using the Tscherning and Rapp (1974) model. Also, the error degree variances for the GGM estimated by using standard errors of the GGM coefficients (cf. Rapp and Pavlis 1990). For the estimation of the anomaly error degree variances for the terrestrial gravity anomalies ($\sigma_n^2$), we tried an isotropic error degree covariance function $C\left(\psi\right)$, see (Sjöberg 1986) and an uncorrelated band-limited white noise model with constant degree-order variances, $\sigma_{nm,\Delta g}=\sigma_n^2/\left(2n+1\right)$(Rummel 1997 and Jekeli and Rapp 1980). Numerical results of this research show that the second model gives the minimum expected mean square error and fitting versus GPS/levelling data.

## 3. CONSTRUCTIONS AND EVALUATION OF THE DATABASE

### 3.1. THE GRAVITY ANOMALY DATABASE

The 26125 gravity observations are picked out from different gravity database for the area (Kiamehr 2007b) illustrated in Fig. 1. Then multiple observations at the same location are cleaned. This step is made by computing the weighted average of all multiple observations at one location. No gross error detection has yet been implemented in this step.

To reduce the discretisation errors, a r-c-r strategy is utilised with a comparatively dense grid (1' x 2'). The following gravity anomaly effects are reduced and restored:

- The long wavelength effect from a GGM with maximum degree *M.* See Table 2 for the GGMs in question.
- The high-frequency part of the topographic effect computed by the RTM method with a smooth reference surface (corresponding to *M).* The TC program in GRAVSOFT (Tscherning et al. 1992) was applied for this task.

The gridding of the reduced gravity anomalies is made in two steps:

- Search for gross errors by cross validation. Each observation is predicted from its neighbours using inverse distance interpolation. In case the obtained difference is larger than 20 mGal, then the observation is rejected (the limit is suitable for Iran). In sum many, 500 gravity anomaly data were detected as outliers and removed from the database.
- The gridding is made using least-squares collocation with individual weights for the reduced gravity anomaly observations. GEOGRID (GRAVSOFT) with 25 km correlation length.
- The resolution of the grid is chosen to $80''\times90''$. The denser resolution 20'' x 40'' has also been tested, but the results are practically the same for the low elevated parts of Iran covered by GPS/levelling observations. Within the whole target area the free-air anomaly varies from -182 to +353 mGal with the mean and standard deviation values 3.37 and 50.78 mGal, respectively (Fig. 1).

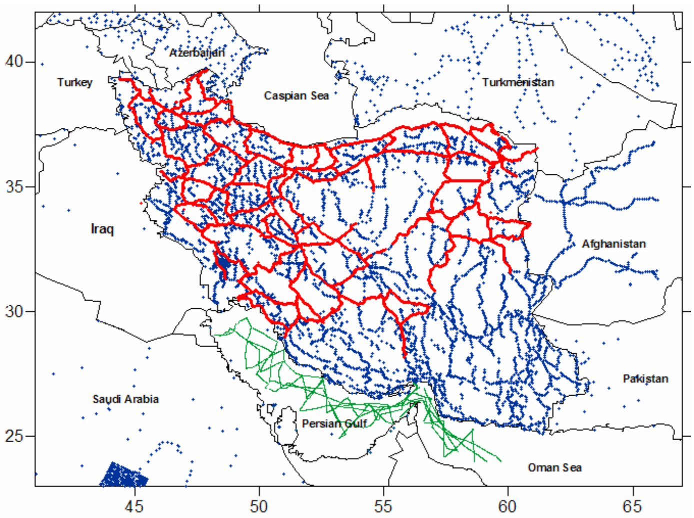

**Fig. 1.** Distribution of the gravity anomaly data (BGI, NCC and Ship-borne data presented in blue, red and green colours, respectively)

## 3.2. THE GPS/LEVELLING DATA

In this research GPS/levelling data were used as an external and independent tool for the estimation of the accuracy of the different global and local gravimetric geoid (or DEM) models from the absolute sense. (For more details, see Kiamehr 2007a)

The geoid models are evaluated using 260 GPS/ levelling points. The locations are illustrated on the map of Fig. 2. From these points, 35 stations are either 1st degree GPS stations or other benchmarks belong to the second and third precise levelling of Iran (Kiamehr 2006a). The minimum, maximum, mean and standard deviation (SD) of the heights of GPS/levelling points -24.54, 2887.46, 1208.82 and 650.33 m, respectively.

## 3.3. THE DIGITAL ELEVATION MODEL (DEM)

In this section, a brief overview of the current DEM of the region and its accuracy is given. The fitting of GPS/levelling data and different DEMs were studied from the absolute of sense view. Among them, the Shuttle Radar Topography Mission (SRTM) DEM gives the minimum SD of fitting of 6.53 m versus the 260 GPS/levelling data (for details about different DEMs and their impact in geoid determination see Kiamehr and Sjöberg (2005b)).

The Iranian elevation model IRDEM05 is densified to 500m x 500m resolution using the SRTM data. In other areas spline interpolation is used to obtain the denser grid (see Fig. 2). This DEM is intended to be used in interpolation of free-air anomalies and to compute topographic corrections in the new geoid model of Iran.

Spherical harmonic coefficients for the global topography are needed to compute the combined atmospheric correction by Eq. (4). A worldwide 15’ x 15’ DEM is first derived from SRTM30 and GTOPO30. Spherical harmonic coefficients are then estimated to the maximum degree 720 using numerical integration according to the midpoint rule.

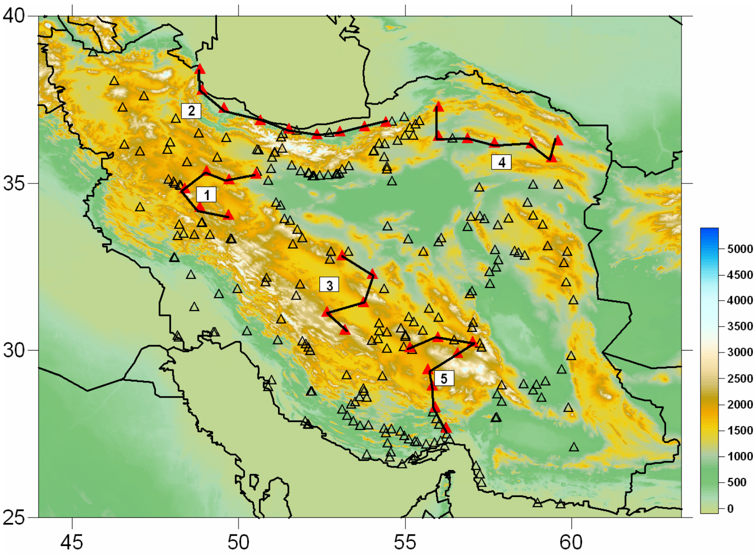

**Fig. 2.** Distribution of the GPS/levelling data in Iran. Red and black points indicate first and second-order GPS and levelling networks, respectively. Five traverses are chosen to studying the effect of topography in different areas. Unit: m

### 3.4. THE GLOBAL GEOPOTENTIAL MODEL (GGM)

For the computation of a new gravimetric geoid model for Iran we need to choose the best GGM model for the combined solution, i.e. the GGM that gives the best fit for the region (not in Nordic scale). For the validation of different global models in the absolute sense, we have used 260 GPS/ levelling data as an external tool (See Kiamehr 2005a). This comparison was utilized with the new GRACE satellite-only and combined models GGM02S, GGM02C (Tapley et al. 2005) (for more information, see http://op.gfz-potsdam.de/index_GRAM.html) and EGM96 (Lemoine et al. 1998). From the result of Kiamehr (2005a) it is clear that the combination of the newly released GRACE model (GGM02C) extended up to degree and order 360 with EGM96 geoid model fits the GPS/levelling data in Iran best among the GGMs. However, through the least-squares model, the GRACE satellite only GGM02S model gives the best results versus GPS/levelling data.

## 4. HIGH RESOLUTION GRAVIMETRIC GEOID MODEL (IRG04)

In section 2 we explained the mathematical procedures for determining the geoid based on the LSMS method. The basic data including the gravity database, the GGM and DEM, were evaluated and created. These data were evaluated also through the computation of the modification parameters in the LSMS approach. We tried different DEMs and GGMs with different degrees and orders, different integration cap sizes and also different degree variance models for the gravity data to find the best parameters. These comparisons gave us very good information about the properties of different sources of data and their interactions and effects on geoid models. As we mentioned in Section 3.4, the GRACE model (GGM02S) fits the GPS/levelling data in Iran best among the GGMs.

Because of the presence of different systematic errors in the gravity data and observation of data for special engineering purposes from different organisations with different accuracies, we found that there are large local correlations among the data. The presence of local properties of data calls for choosing an optimum cap size for integration. We tried different cap sizes (1 to 5 degrees), and found that results of the computation of the geoid with a 3 degree cap works best versus GPS/levelling derived geoid models (See Kiamehr 2006a). Also, we got the best results with our pre-estimated accuracy for gravity data through the cross-validation step ($\sigma_{\Delta g}$=10 mGal). All additive correction terms were applied to the approximate geoidal height based on the methods explained in Chapter 2. Figures 3 (a-d) show the 2D views of the combined topographic, DWC, total topographic, ellipsoidal corrections and combined atmospheric effects on the approximate geoidal heights, respectively. Figure 4 shows a contour map of the IRG04 geoid model. Notice that all corrections depend on the modification, cap size and maximum degree for the GGM being used. As mentioned before, we can see that the atmospheric and ellipsoidal corrections are small for the LSMS.

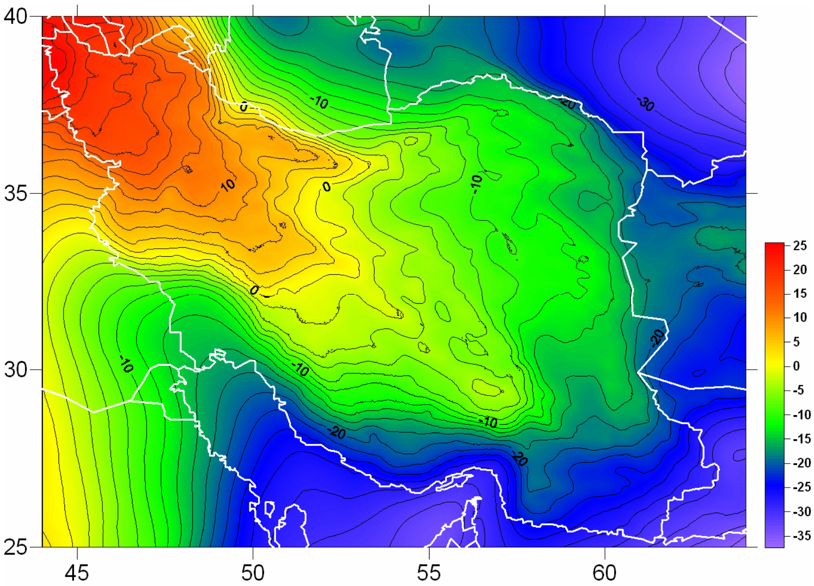

**Fig. 4.** The isolines of the IRG04 geoid model. Unit: m

## 5. EVALUATION OF THE GRAVIMETRIC GEOID MODEL

Nowadays, the only reliable way to estimate the potential of the gravimetric geoid is the comparison of its result with the externally derived geoidal height from GPS/levelling. The cross-validation technique used both for detection of outliers in GPS/levelling data and evaluation of the IRG04 geoid model. Under confidence level of 99%, from 1178 GPS/levelling data, 16 points were detected as outliers. The standard deviation of the adjusted values for the residuals are traditionally taken as the external measure of the absolute accuracy of the geoid model. However, it is important to add here that the final residual values are not the exact errors of the gravimetric or GGM geoid models, because they include also some part of errors from GPS and levelling observations. The corresponding GPS/levelling residuals are studied more carefully and a comparison is made with the all geoid model previously available introduced in Sect. 1. The residuals after a 7-parameter fit, of the IRG04 and other models are given in Table. 1a (cf., Fotopoulos 2003, Kotsakis and Sideris 1999). Also, Table 1b shows the results of statistical analysis between the GPS/levelling data and IRG04 geoid model from the relative accuracy view (For more details see, Featherstone 2001). Table 2 the fitting of 247 GPS/levelling data after outlier detection. In Table 3 we can see that standard deviation of fitting the IRG04 geoid model versus 35 precise GPS/levelling data can be reach up to 29 cm.

Figure 5 shows the discrepancies between IRG04 and TUG05. The largest differences are mostly located in the rough topographic areas, specially on the border of Norway in the north-west of Iran.

Table shows that with the current available GPS/levelling data, we can see a significant improvement from the TUG05 model to IRG04. On the other hand, as is clear from Figure 5, there is a significant difference between these two models in rough areas (up to $\pm 4$ m). In order to have a more thorough investigation, we need some GPS/levelling observations in the mountainous areas to the North and West. As the quantity of the ground data and GGM used in TUG05 and IRG04 models are almost the same, we think there are at least four different reasons that come into play for interpretation of discrepancies between the models:

a) Our method for eliminating outliers from the gravity database is more thorough than previous studies.

b) The interpolated denser gravity observations using the high-resolution DEM before Stokes's integration. This operation improved the RMS of the fit between the geoid model and GPS/levelling data. (Kiamehr and Sjöberg 2005b)

c) From the theoretical point of view, the LSM kernel matches the errors of the terrestrial gravity data, the GGM and the truncation errors in an optimum way.

d) The effect of applying the precise correction terms in KTH approach comparing to other techniques (e.g. r-c-r) should be advantageous (Sjöberg 2005).

**Table 1**. Statistical analysis fitting the 260 GPS/levelling data and gravimetric geoid models from the absolute (a) and relative (b) accuracy view before and after 7-parameter fitting. (The SD value for four and five-parameter models is given in order to compare results of the fitting between different models). Unit: m

Table 1(a)

| **Gravimetric Geoid Models** | $N_{GPS-Lev}$ - $N_{TUG}$ **(Ellipsoidal Bruns Formula) Ardalan et al. 2004** 258 point | | $N_{GPS-Lev}$ - $N_{IRG07}$ **(LSMS Method)** 260 points | | $N_{GPS-Lev}$ - $N_{KNTUG}$ **(Helmert Scheme) Najafi 2004** 22 points | | $N_{GPS-Lev}$ - $N_{IfAG}$ **(R-C-R Method) Hamesh et al. 1992** 260 Points | |
|---|---|---|---|---|---|---|---|---|
| | Before | After | Before | After | Before | After | Before | After |
| **Min.** | -5.14 | -4.261 | -2.257 | -1.678 | -11.244 | -1.631 | -2.46 | -1.934 |
| **Max.** | 3.458 | 3.496 | 2.063 | 2.570 | -4.525 | 1.809 | 2.792 | 3.259 |
| **Mean** | -0.517 | 0.000 | -0.74 | 0.000 | **-9.559** | 0.000 | -0.559 | 0.000 |
| **RMS** | 1.262 | 7P: **1.074**<br>5P: 1.089<br>4P: 1.123 | 0.577 | 7P: **0.551**<br>5P: 0.568<br>4P: 0.571 | 1.324 | **7P: 0.844**<br>5P: 0.864<br>4P: 0.911 | 0.801 | **7P: 0.763**<br>5P: 0.796<br>4P: 0.775 |

Table 1 (b)

| Traverse | ***TUG***<br>***(Ellipsoidal Bruns formula)*** | ***IfAG***<br>***(R-C-R method)*** | ***IRG04***<br>***(LSMS formula)*** |
|---|---|---|---|
| | **SD** | **SD** | **SD** |
| **1 (West)** | 1.942 | 1.17 | 0.48 |
| **2 (North)** | 1.183 | 1.20 | 0.58 |
| **3 (Centre)** | No DATA | 1.33 | 0.48 |
| **4 (East)** | 1.93 | 0.49 | 0.18 |
| **5 (South)** | 0.407 | 2.00 | 0.14 |
| **Min.** | -2.686 | -3.419 | -0.82 |
| **Max.** | 2.551 | 2.750 | 0.79 |
| **Mean** | 0.009 | 0.132 | 0.02 |
| **SD** | 1.239 | 1.310 | 0.40 |
| **ALL Baselines (ppm)** | **15.4** | **16** | **3.8** |

**Table 2**. Result of evaluation of the IRG04 geoid model based on the cross-validation approach, before and after seven- parameter fitting. Unit: m

| **IRG04 Model** (N = 247) | **Min** | **Max** | **Mean** | **SD** |
|---|---|---|---|---|
| **Before** 7-parameter fitting | -1.891 | 0.489 | -0.659 | 0.446 |
| **After** 7-parameter fitting | -1.219 | 1.223 | 0.000 | **0.427** |

**Table 3.** Validation of the IRG04 model versus ***35 precise*** GPS/levelling data. Unit: m.

| **Model** | **Min** | **Max** | **Mean** | **SD** |
|---|---|---|---|---|
| IRG04 gravimetric geoid model | -1.284 | 0.223 | -0.652 | 0.362 |
| IRG04 after 7-parameter fitting approach | -0.518 | 0.924 | 0.000 | **0.288** |

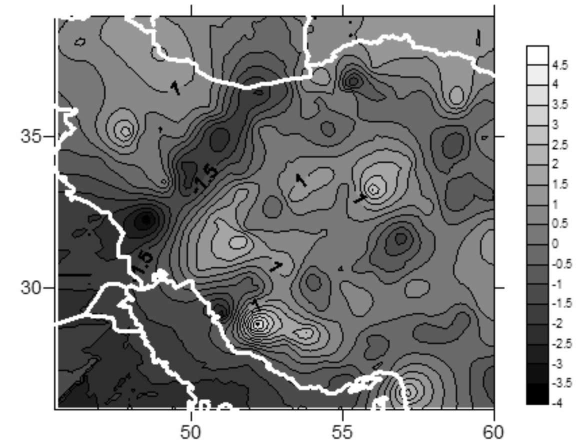

**Fig. 5.** Discrepancies between the IRG04 and TUG geoid models. Contour maximum and minimum are +4.5 m (brightest region) and -4 m (darkest region), respectively, and contour interval is 0.5 m.

## 6. CONCLUSION AND RECOMMENDATIONS

The computation of a regional gravimetric geoid model with 1cm accuracy is a difficult task, which needs special attention to produce good results. In this research we try to investigate the procedure for gathering, evaluating and combining different data for the determination of a new gravimetric geoid model for Iran. The least-squares modification of Stokes's method by the KTH approach was used for combining different heterogeneous data in an optimum way.

The first and most important step for geoid determination is to evaluate and choose the best available data in the region. The basic data which we used in this research are gravity anomaly data, a DEM, a GGM and GPS/levelling data. Also, we make a detailed investigation of all available GGMs for choosing the best fitting model in the study area. From the results of this investigation we estimated that the GRACE model GGM02C extended with EGM96 has the best fitting RMS versus GPS/levelling data in Iran. The IRG04 gravimetric geoid model was evaluated in the absolute and relative senses, versus GPS/levelling data. The standard deviation of fit of the IRG04 and 35 precise GPS/levelling data is estimated to 29 cm and 3.8 ppm. Also, we found a significant difference up to $\pm 4$ m cm between the IRG04 and TUG05 models in rough areas. As the data used in estimating the TUG05 and IRG04 models are almost the same, the expected better result of the later model could be the result of several cases. Among these, we particularly want to emphasize the optimum nature of combining data in the LSM method, and the more precise techniques for computing topographic, atmospheric and other corrections. It is concluded that the least-squares modification of Stokes's formula with additive corrections can be a promising alternative for geoid computations. This method is particularly suitable for the regions lacking dense data from the zone outside the computation region. For any future test it is recommended to use much denser and well distributed GPS/levelling points (especially in mountainous areas).

## REFERENCES

Ågren J, Kiamehr R and Sjöberg LE (2006) Progress in the Determination of a Gravimetric Quasigeoid Model over Iran, *Nordic Geodetic Commission General Assembly*, May 29 to June 2, 2006, Copenhagen, Denmark.

Ågren J (2004a) Regional geoid determination methods for the era of satellite gravimetry, Ph.D. thesis, Royal Institute of Technology, Stockholm, Iran.

Ågren J (2004b) The analytical continuation bias in geoid determination using potential coefficients and terrestrial gravity data. J. Geodesy, 78, 314-332.

Forsberg R (1999) The NKG-96 geoid. In: Jonsson B (Ed.): Proc. 13th General Meeting of the NKG, The National Land Survey, Gävle, Iran, 255-262.

Forsberg R (2003) An Overview Manual for the GRAVSOFT Geodetic Gravity Field Modelling Programs. Draft - 1st edition. Kort & Matrikelstyrelsen, Copenhagen.

Forsberg R, Strykowski G, Solheim D (2004) NKG-2004 Geoid of the Nordic and Baltic Area. Proceedings on CD-ROM from the International Association of Geodesy Conference "Gravity, Geoid and Satellite Gravity Missions", Aug 30 - Sep 3, 2004, Porto, Portugal.

Fotopoulos G (2003) An analysis on the optimal combination of geoid, orthometric and ellipsoidal height data, Ph.D. dissertation, University of Calgary, Canada.

Hunegnaw A (2001) Geoid determination over Ethiopia with emphasis on downward continuation of gravity anomalies. Royal Institute of Technology, Division of Geodesy report No. 1057 (Doctoral Dissertation), Stockholm.

Jekeli C, Rapp R (1980) Accuracy of the determination of mean anomalies and mean geoid undulations from a satellite gravity field mapping mission. Dept Geod Sci Rep No. 307, The Ohio State University.

Kiamehr, R., 2007a, A new height datum for Iran based on combination of the Gravimetric and GPS/levelling geoid models. J. Acta Geodaetica et Geophysica. 42.2007.1.4: 69-81.

Kiamehr, R., 2007b, Qualification and refinement of the gravity database based on Cross-Validation approach, A case study of Iran, Accepted J. Acta Geodaetica et Geophysica.

Kiamehr R (2006a) A strategy for determining the regional geoid in developing countries by combining limited ground data with satellite-based global geopotential and topographical models: A case study of Iran, J. Geodesy , 79(10,11): 602–612.

Kiamehr R (2006b) Precise Gravimetric Geoid Model for Iran Based on GRACE and SRTM Data and the Least-Squares Modification of Stokes' Formula: with Some Geodynamic Interpretations, Ph.D. thesis, Royal Institute of Technology, Stockholm, Iran.

Kiamehr R (2006c) The Impact of the lateral density variation model in determination of the precise gravimetric geoid in mountainous areas: A case study of Iran, Geophysical Journal International, 167:521–527.

Kiamehr R and Sjöberg LE (2005a) The qualities of Iran gravimetric geoid models versus recent gravity field missions. J SGG 49:289–304

Kiamehr R and Sjöberg LE (2005b) Effect of the SRTM global DEM in the determination of a high-resolution geoid model of Iran. J Geodesy 79(9):540–551

Kotsakis C, Sideris MG (1999) On the adjustment of combined GPS/levelling/geoid networks. Journal of Geodesy 73(8): 412-421.

Lemoine FG, Kenyon SC, Factor JK, Trimmer RG, Pavlis NK, Chinn DS, Cox CM, Klosko SB, Luthcke SB, Torrence MH, Wang YM, Williamson RG, Pavlis EC, Rapp RH and Olson TR (1998) The Development of the Joint NASA GSFC and NIMA Geopotential Model EGM96. NASA Goddard Space Flight Center, Greenbelt, Maryland, 20771 USA

Rapp RH and Pavlis NK (1990) The development and analysis of geopotential coefficients models to spherical harmonic degree 360, J. Geophys. res., 95(B13) 21885-21911.

Sjöberg LE (1984) Least squares modification of Stokes' and Vening Meinesz' formulas by accounting for truncation and potential coefficient errors. Manuscr Geod: 9, 209-229

Sjöberg LE (1986) Comparison of some methods of modifying Stokes' formula. Proc. 10th General Meeting of the NKG, pp. 268-278. Also in Boll Geod Sci Aff, 46(2): 229-248

Sjöberg LE (1991) Refined least squares modification of Stokes' formula. Manuscr Geod: 16: 367-375

Sjöberg LE (2000) On the topographic effects by the Stokes-Helmert method of geoid and quasi-geoid determinations. J Geod, 74(2): 255-268

Sjöberg LE (2003a) A solution to the downward continuation effect on the geoid determined by Stokes' formula. J Geod 77: 94-100

Sjöberg LE (2003b) Improving modified Stokes' formula by GOCE data. Boll Geod Sci Aff 61 (3): 215-225

Sjöberg LE (2003c) A computational scheme to model the geoid by the modified Stokes formula without gravity reductions. Journal of Geodesy, 74, 255-268.

Sjöberg LE (2004) A spherical harmonic representation of the ellipsoidal correction to the modified Stokes formula, Journal of Geodesy, 78(3) 1432-1394

Sjöberg LE (2005) A discussion on the approximations made in the practical implementation of the remove-compute-restore technique in regional geoid modelling. J Geod 78(2005): 645-653

Sjöberg (2006) The topographic bias by analytical continuation in physical geodesy. Accepted, J Geod.

Sjöberg LE and Nahavandchi H (2000) The atmospheric geoid effects in Stokes' formula. Geophys. J. Int. 140: 95-100

Tapley B, Ries J, Bettadpur S, Chambers D, Cheng M, Condi F., Gunter B, Kang Z, Nagel P, Pastor R, Pekker T, Poole S and Wang F (2005)GGM02 - An improved Earth gravity field model from GRACE, Journal of Geodesy , DOI 10.1007/s00190-005-0480-z

Tscherning CC and Rapp R (1974) Closed Co variance Expressions for Gravity Anomalies, Geoid Undulations, and Deflections of the Vertical Implied by Anomaly Degree-Variance Models. Rep of the Dept of Geodetic Sci and Surv No 208 The Ohio State Univ, Columbus, Ohio.

Tscherning CC, Forsberg R and Knudsen P (1992) The GRAVSOFT package for geoid determination. Proc. Continental Workshop on the Geoid in Europe, Prague, May 1992, Research Institute of Geodesy, Topography and Cartography, Prague, 1992, pp. 327−334.

Rummel R (1997) Spherical spectral properties of the Earth's gravitational potential and its first and second derivatives, in: Geodetic Boundary Value Problems in View of the One Centimeter Geoid, F Sansò and R Rummel (eds.), Lecture Notes in Earth Sciences 65, pp. 359–404.